Constructing SAS Contrast/Estimate Statements

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In the case of single factors, the coefficients for estimate/contrast statements to compare means are easily specified. Coefficients for mean contrasts involving interaction of two or three factors are more complex to construct. The following should help guide you through the process to obtain the coefficients necessary in order to compare interaction means.

Step 1. Identify the order of levels within each factor. This will either be alpha-numeric **or** in order of first occurrence in the datafile if *order=data* is specified. Either review the order of levels given in the first page of the Proc Mixed/GLM output or request the lsmeans – the order that is given is the order of levels within each factor.

Step 2. Identify the order of factors. This is specified by the order they have been listed in the *class* statement. If you have three factors, A, B, and C, and the *class* statement is: *class* A B C; then for interactions, the A*B*C interaction means will be arranged as ABC. If instead you have them listed as *class* A C B, then the A*B*C means will be arranged as ACB.

Step 3. Identify the specific means, or group of means, you want to contrast.

Assume you have an experiment with Factor A with two levels (a1 & a2), Factor B with three levels (b1, b2, & b3) and Factor C with four levels (c1, c2, c3, & c4).

Factors		
A	B	C
a1 a2	b1 b2 b3	c1 c2 c3 c4

The following examples assume the class statement specifies the order of factors to be A-B-C:

class A B C;

I. EXAMPLE 1. Contrast of means within one factor.

Contrast of the mean of b1 vs the mean of b2 (averaged over all levels of A & C).

estimate ' b1 vs b2' B 1 -1 0;

The sum of the coefficients must be zero. If more means are involved in the comparison, either use decimal coefficients or use whole numbers and the */divisor=n* option. For example, comparing the mean of b1 & b2 to the mean of b3 could be specified either as **estimate 'b1&b2 vs b3' B 0.5 0.5 -1;** or **'b1&b2 vs b3' B 1 1 -2 /divisor=2;** The latter method is preferable as it avoids situations such as long and repeating decimals.

II. EXAMPLE 2. Contrast of two means at a set value of another factor.

Contrast of the mean of b1 vs the mean of b2 when the level of A is a1.

i) Create a table with the levels of A as rows and the levels of B as columns. Beside/below these, enter the coefficients associated with the Comparison of interest. In this case it is b1 vs b2 (1 -1 0), and for when the level of A is a1 (1 0).

			B			
			b1	b2	b3	
			1	-1	0	
A	a1	1				
	a2	0				

ii) Multiply the coefficients in each row-column combination and enter the value in the respective cell in the centre of the table.

			B			
			b1	b2	b3	
			1	-1	0	
A	a1	1	**1**	**-1**	**0**	
	a2	0	**0**	**0**	**0**	

iii) Compute the total across each rows and within each column of the cells computed in step ii). Also compute the grand total.

			B			
			b1	b2	b3	
			1	-1	0	
A	a1	0	**1**	**-1**	**0**	**0**
	a2	1	**0**	**0**	**0**	**0**
			1	**-1**	**0**	0

The coefficients can be obtained from the above table.

1. The intercept value is the grand total.
   In this case it is 0, so the intercept drops out of the computation.
2. The row totals (shaded) give the coefficients for the A factor.
   In this case they are all 0 so the A factor drops out of the computation.
3. The column totals (shaded) give the coefficients for the B factor.
   In this case they are 1 -1 0
4. The 'computed' cells are the coefficients for the AxB factor; these are in bold. Note that you collate the coefficients by reading across each row.
   In this case, the AxB interaction coefficients are 1 -1 0 0 0 0

The estimate (or contrast) statement for comparing the mean of b1 vs mean of b2 when the level of A is a1 would be specified as:

estimate ' b1 vs b2 when A=a1' B 1 -1 0 A*B 1 -1 0 0 0 0;

III. EXAMPLE 3. Contrast of two means at a set value of two other factors.

Contrast of the mean of b1 vs the mean of b2 when the level of A is a1 <u>and</u> the level of C is c1.

With three factors, there are eight possible terms that may be involved in specifying an estimate/contrast statement; namely, the intercept, A, B, C, AxB, AxC, BxC, and AxBxC interactions.

i) With three factors, this will involve main, two-factor, and three-factor interaction coefficients. Begin by creating a table with the levels of A as rows and the levels of B as columns. Continue as for Example 2 to obtain the coefficients for the AxB interaction. However, don't bother summing the rows and columns in this step.

			B		
			b1	b2	b3
			1	-1	0
A	a1	1	**1**	**-1**	**0**
	a2	0	**0**	**0**	**0**

ii) Now create a table of the AxB interaction coefficients against C (**AxB against C**).

				C			
				c1	c2	c3	c4
				1	0	0	0
A	a1	b1	1				
		b2	-1				
		b3	0				
	a2	b1	0				
		b2	0				
		b3	0				

iii) Multiply the coefficients in each row-column combination and enter the value in the respective cell in the centre of the table.

				C			
				c1	c2	c3	c4
				1	0	0	0
A	a1	b1	1	**1**	**0**	**0**	**0**
		b2	-1	**-1**	**0**	**0**	**0**
		b3	0	**0**	**0**	**0**	**0**
	a2	b1	0	**0**	**0**	**0**	**0**
		b2	0	**0**	**0**	**0**	**0**
		b3	0	**0**	**0**	**0**	**0**

iv) Add across the rows and along the columns of the cells computed in step iii). Also compute the grand total.

				C				
				c1	c2	c3	c4	
				1	0	0	0	
A	a1	b1	1	**1**	**0**	**0**	**0**	**1**
		b2	-1	**-1**	**0**	**0**	**0**	**-1**
		b3	0	**0**	**0**	**0**	**0**	**0**
	a2	b1	0	**0**	**0**	**0**	**0**	**0**
		b2	0	**0**	**0**	**0**	**0**	**0**
		b3	0	**0**	**0**	**0**	**0**	**0**
				0	**0**	**0**	**0**	0

This table gives the coefficients for the **AxBxC** interaction, the **AxB** interaction (last column), the **C factor** (last row), and the **intercept** (grand total).

For this example, the AxBxC interaction coefficients are 1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 the AxB interaction coefficients are 1 -1 0 0 0 0 and the C factor and the intercept drop out since all of their coefficients are zero.

However, for a three-factor, there are two additional tables that need to be computed: The AxC against B and BxC against A.

AxC against B.

i) Obtain the AxC coefficients:

			C				
			c1	c2	c3	c4	
			1	0	0	0	
A	a1	1	**1**	**0**	**0**	**0**	
	a2	0	**0**	**0**	**0**	**0**	

ii) Obtain the AxC against B coefficients:

				B			
				b1	b2	b3	
				1	-1	0	
A	a1	c1	1	**1**	**-1**	**0**	**0**
		c2	0	**0**	**0**	**0**	**0**
		c3	0	**0**	**0**	**0**	**0**
		c4	0	**0**	**0**	**0**	**0**
	a2	c1	0	**0**	**0**	**0**	**0**
		c2	0	**0**	**0**	**0**	**0**
		c3	0	**0**	**0**	**0**	**0**
		c4	0	**0**	**0**	**0**	**0**
				1	**-1**	**0**	0

This table gives the coefficients for the **AxC** interaction (last column) and the **B factor**.

In this example, the AxC interaction drops out, the B factor remains with coefficients 1 -1 -0.

BxC against A.

i) Obtain the BxC coefficients:

			C			
			c1	c2	c3	c4
			1	0	0	0
B	b1	1	**1**	**0**	**0**	**0**
	b2	-1	**-1**	**0**	**0**	**0**
	b3	0	**0**	**0**	**0**	**0**

ii) Obtain the BxC against A coefficients:

				A		
				a1	a2	
				1	0	
B	b1	c1	1	**1**	**0**	**1**
		c2	0	**0**	**0**	**0**
		c3	0	**0**	**0**	**0**
		c4	0	**0**	**0**	**0**
	b2	c1	-1	**-1**	**0**	**-1**
		c2	0	**0**	**0**	**0**
		c3	0	**0**	**0**	**0**
		c4	0	**0**	**0**	**0**
	b3	c1	0	**0**	**0**	**0**
		c2	0	**0**	**0**	**0**
		c3	0	**0**	**0**	**0**
		c4	0	**0**	**0**	**0**
				0	**0**	0

This table gives the coefficients for the **BxC** interaction (last column) and the **A factor**. In this example, the A factor drops out, the BxC interaction remains with coefficients 1 0 0 0 -1 0 0 0 0 0 0 0

Combining the information from the AxBxC, and the AxC against B, and BxC against A tables, the intercept, A, C, & AxC terms drop out. The estimate statement for this particular comparison requires coefficients for B, AxB, BxC, and AxBxC.

The estimate (or contrast) statement for comparing the mean of b1 vs mean of b2 when the level of A is a1 and the level of C is c1 would be specified as:

```
estimate   ' b1 vs b2 when A=a1 & C=c1'   B    1 -1 0   A*B   1 -1 0 0 0 0
           B*C  1 0 0 0 -1 0 0 0 0 0 0 0
           A*B*C  1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
```

[Note: The trailing zeros can be dropped in the above coefficients.]

Similar estimates statements for other levels can also be generated by shifting the 1 & -1 to the appropriate location within the sequence. For example, if factor C was time, the comparison at time=4 (the endpoint) would be:

```
estimate   ' b1 vs b2 when A=a1 & C=c4'   B    1 -1 0   A*B   1 -1 0 0 0 0
           B*C 0 0 0 1 0 0 0 -1 0 0 0 0
           A*B*C   0 0 0 1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
```